

6주차 3차시 : 다중회귀분석(분산 추정 및 결정계수)

1. 분산 추정
2. 결정계수

1. 분산 추정

(1) 교란항의 분산

$$\hat{\sigma}_u^2 = \frac{e'e}{n-k}$$

$$e'e = Y'Y - 2\hat{B}'X'Y + \hat{B}'X'X\hat{B} \quad \leftarrow \text{(최소자승법의 목적함수에서)}$$

$$= Y'Y - 2\hat{B}'X'X\hat{B} + \hat{B}'X'X\hat{B} \quad \leftarrow Y = X\hat{B} \text{ (정규방정식에서)}$$

$$= Y'Y - \hat{B}'X'X\hat{B} \quad \text{또는} \quad Y'Y - \hat{B}'X'Y$$

(2) 회귀계수의 분산

$$\hat{B} = (X'X)^{-1}X'Y$$

$$= (X'X)^{-1}X'(XB + U)$$

$$= (X'X)^{-1}X'XB + (X'X)^{-1}X'U$$

$$= B + (X'X)^{-1}X'U$$

$$\hat{B} - B = (X'X)^{-1}X'U$$

$$\begin{aligned} \text{Var}(\hat{B}) &= E[(\hat{B} - E(\hat{B}))(\hat{B} - E(\hat{B}))'] \\ &= E[(\hat{B} - B)(\hat{B} - B)'] \\ &= E[(X'X)^{-1}X'UU'X(X'X)^{-1}] \quad \leftarrow \quad \because (A^{-1})' = (A')^{-1} \\ &= (X'X)^{-1} X'E(UU')X(X'X)^{-1} \\ &= \sigma_u^2(X'X)^{-1} \end{aligned}$$

회귀계수의 분산-공분산행렬 (variance-covariance matrix)

$$\begin{aligned} \text{Var}(\hat{B}) &= \begin{bmatrix} \text{Var}(\hat{\beta}_1) & \text{Cov}(\hat{\beta}_1\hat{\beta}_2) & \text{Cov}(\hat{\beta}_1\hat{\beta}_3) & \cdots & \text{Cov}(\hat{\beta}_1\hat{\beta}_k) \\ \text{Cov}(\hat{\beta}_2\hat{\beta}_1) & \text{Var}(\hat{\beta}_2) & \text{Cov}(\hat{\beta}_2\hat{\beta}_3) & \cdots & \text{Cov}(\hat{\beta}_2\hat{\beta}_k) \\ \vdots & & \ddots & & \vdots \\ \text{Cov}(\hat{\beta}_k\hat{\beta}_1) & \text{Cov}(\hat{\beta}_k\hat{\beta}_2) & \text{Cov}(\hat{\beta}_k\hat{\beta}_3) & \cdots & \text{Var}(\hat{\beta}_k) \end{bmatrix} \\ &= \sigma_u^2(X'X)^{-1} \end{aligned}$$

2. 결정계수(coefficient of determination)

$$Y'Y = (\hat{Y} + e)'(\hat{Y} + e)$$

$$= (X\hat{B} + e)'(X\hat{B} + e)$$

$$= (\hat{B}'X' + e')(X\hat{B} + e)$$

$$= \hat{B}'X'X\hat{B} + \hat{B}'X'e + e'X\hat{B} + e'e$$

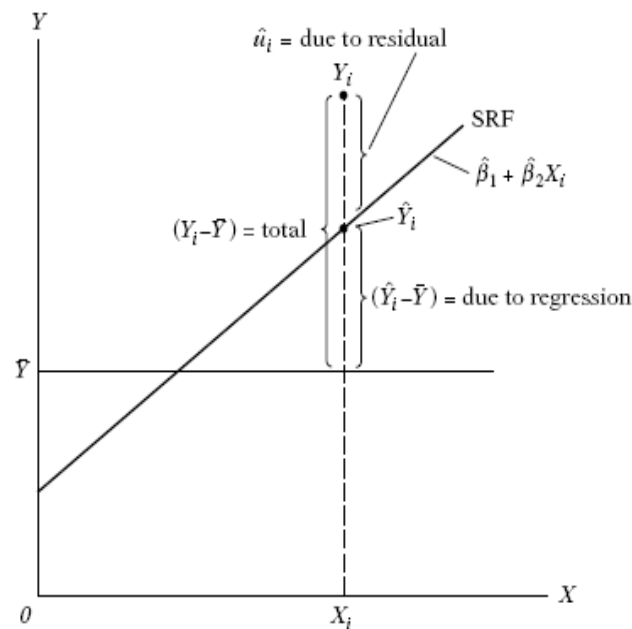
$$= \hat{B}'X'X\hat{B} + 2\hat{B}'X'e + e'e$$

$$= \hat{B}'X'X\hat{B} + e'e \leftarrow \because \hat{B}'X'e = \hat{B}'X'(Y - X\hat{B}) = \hat{B}'(X'Y - X'X\hat{B}) = 0$$

$\because X'X\hat{B} = X'Y$ (정규방정식)

$$R^2 = \frac{\hat{B}'X'X\hat{B}}{Y'Y} \quad \text{단, } \bar{Y}=0 \text{ 일 때}$$

$$R^2 = \frac{\hat{B}'X'X\hat{B} - n\bar{Y}^2}{Y'Y - n\bar{Y}^2} \quad \text{단, } \bar{Y} \neq 0 \text{ 일 때}$$



(예제-계속)

$$e'e = Y'Y - \hat{B}'X'X\hat{B}$$

$$= [1 \quad 1 \quad 2 \quad 3] \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - [-4.25 \quad 1.5 \quad 2] \begin{bmatrix} 4 & 8 & 6 \\ 8 & 18 & 11 \\ 6 & 11 & 10 \end{bmatrix} \begin{bmatrix} -4.25 \\ 1.5 \\ 2 \end{bmatrix}$$

$$= 15 - 14.75$$

$$= 0.25$$

$$\therefore \hat{\sigma}_u^2 = \frac{0.25}{4-3} = 0.25$$

```
> ypy=t(y)%*%y
> ypy
      [,1]
[1,]    15
> bpxpxb=t(beta)%*%t(x)%*%x)%*%beta
> bpxpxb
      [,1]
[1,] 14.75
> epe=ypy-bpxpxb
> epe
      [,1]
[1,] 0.25
> sigusq=epe/(n-3)
> sigusq
      [,1]
[1,] 0.25
> |
```

$$\therefore R^2 = \frac{\hat{B}'X'X\hat{B} - n\bar{Y}^2}{Y'Y - n\bar{Y}^2} = \frac{14.75 - 4(1.75)^2}{15 - 4(1.75)^2} = \frac{2.5}{2.75} = 0.909$$

$$\therefore \text{Var}(\hat{B}) = \sigma_u^2(X'X)^{-1}$$

$$= (0.25) \begin{bmatrix} 14.75 & -3.5 & -5 \\ -3.5 & 1 & 1 \\ -5 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3.6875 & -0.875 & -1.25 \\ -0.875 & 0.25 & 0.25 \\ -1.25 & 0.25 & 0.5 \end{bmatrix}$$

$$\text{se}(\hat{B}) = \begin{bmatrix} \sqrt{3.6875} \\ \sqrt{0.25} \\ \sqrt{0.5} \end{bmatrix}$$

```
> rsq<-(bpxpxb-n*mean(y)^2)/(ypy-n*mean(y)^2)
> rsq
      [,1]
[1,] 0.9090909
>
> varcov<-0.25*xpxinv
> varcov
      [,1] [,2] [,3]
[1,] 3.6875 -0.875 -1.25
[2,] -0.8750 0.250 0.25
[3,] -1.2500 0.250 0.50
> se<-sqrt(diag(varcov))
> se
[1] 1.9202864 0.5000000 0.7071068
> |
```