Japanese Yen Behavior since 1980*

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The objective of this paper is to investigate the Japanese real Yen behavior during the Japan’s economically turbulent period since 1980. Using empirical specification of the real Yen following Obstfeld (1993), we have found the real Yen does not follow a trend stationary process, but a unit root process over some sample period since 1980. In the second section of this paper, we have deeply extended Obstfeld (1993)'s approach in showing that the downward trend of real interest rate can be attributed to the appreciation of real currency. Departing from real factors of production or consumption, we have emphasized nominal factors in influencing the real Yen behavior. To show this, we have set up the real interest parity condition and also elicited some testable restrictions: Even though we find no evidence that the real Yen was affected by Japanese money supply over some sub sample period, the real interest parity condition does hold surprisingly over the whole sample period: Relative easing of Japanese domestic money supply causes the real Yen to depreciate while its relative tightened money causes the real Yen to appreciate. Thus, some nominal monetary factors are influential in the behavior of real Yen besides real shocks emphasized in section 1 and section 2.1.

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Keywords: Japanese yen, monetary policy, real exchange rate, real interest parity condition, unit root test

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1. INTRODUCTION

Empirical studies of purchasing power parity using long data series have shown that real exchange rates among the major industrialized countries tend to converge to the long-run level. These studies, which typically examine long time series over one hundred years, reject the null hypothesis of a unit root in real exchange rates by employing the standard Dickey-Fuller unit root test. The convergence of real exchange rates to the long run level may not hold when the data series employed for empirical studies are restricted to short run — especially for floating nominal exchange rates. As Mussa (1986) and Grilli and Kaminski (1991) have noted, the stochastic process for real exchange rates may vary depending on the exchange rate regime of fixed and floating nominal exchange rates.

In accordance with the empirical evidence of the convergence of real exchange rate to the long run level, Obstfeld (1993) showed that the real yen was trend stationary and, therefore, was not a unit root process during the period 1951-1988. The obvious question is whether the trend stationary real yen has persisted during the bubble economy and subsequent stagnation since 1980: The remarkable pattern of real yen is that it steadily appreciates from the early 1970s when the fixed exchange rate regime collapsed until 1995, the year when appreciation trend of real yen changes to real depreciation until the financial crisis 2007, which originated from the U.S.

This paper explores the real Japanese Yen behavior in the turbulent period of Japan, since 1980. The simple reason for the selection of this period is that the yen experienced epic gyrations over the last three decades. The upward trend of Japanese yen real appreciation accelerated after the 1985

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1) See, for example, Edison (1987), Frankel (1986), and Lothian and Taylor (1996). Using panel unit root test, Aminifard and Tayebi (2011) supported PPP hypothesis within (Selected) East Asian countries and Iran.

2) Obstfeld’s paper (1993) was originally written as a comment on Marianne Baxter’s paper titled “Real Exchange rates and Real Interest Rates and Real Interest Differentials: Have We Missed the business cycle Relationship,” Journal of Monetary Economics (1994).

3) Obstfeld (2011) documented some episodes of Japanese yen, related to Japanese monetary policy, over the period 1985-2008. He labeled this period as 'Times of Trouble of Yen'.
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The yen’s abrupt ascent was accompanied by the escalation of asset prices of real estate and stocks. This asset price bubble occurred in 1986, but it ended in 1991. After the collapse of asset bubble, the Japanese economy experienced stagnant economic growth along with serious banking problems.

In section 1 following Obstfeld (1993) we theoretically derive the real exchange in terms of factor productivity parameters by focusing on the production side. Using the empirical specification of the real exchange rate, we investigate the Japanese real exchange rate behavior since 1980. Overall impression of the Japanese real yen is that it does not follow a trend stationary process, but a unit root process over some sample period since 1980. In section 2 following Obstfeld (1993), we have deeply examined the consumption side — particularly, relative consumption of tradables over non-tradables, and the real interest rate behavior just in the framework of real side of consumption and production. Departing from real factors of production or consumption, we have emphasized nominal factors in influencing the real Yen behavior. To show this, we have set up the real interest parity condition and also elicited some testable restrictions. Even though we found no evidence that the real Yen was not affected by Japanese money supply over some sub sample period, the real interest parity condition does hold surprisingly over the whole sample period: Relative easing of Japanese domestic money supply causes the real Yen to depreciate while its relative tightened money causes the real Yen to appreciate. This implies that Japanese monetary policy is helpful in stabilizing Japanese Economy during the time of troubles since 1980.

Han and Lee (2010) present a guide line for a step-by-step to form a monetary union in East Asia. Non-stationary Japanese real Yen implies that real shocks such as productivity shocks were dominant since 1980. It is doubtful that optimal currency union in East Asia can be formed in the presence of non-stationary Japanese real Yen.
2. MODELING DETERMINISTIC OR STOCHASTIC TRENDS IN REAL EXCHANGE RATES

2.1. Model

Focusing only on the production side of the economy, Obstfeld (1993) set up the model for the real exchange rate. We slightly modify his model, particularly in the stochastic process of productivity in non-tradable goods. By doing this, we elicit empirical specification of real exchange movements. Initially, we develop the model without stochastic features. In the end, they will be added. We consider a small open economy, in the sense that some prices are given parametrically. There are two sectors of goods, tradables and non-tradables. Capital and labor are used to produce these goods. Tradable goods are priced in world markets, while the price of non-tradables is determined at home country. The cost of transforming one unit of tradable good into one unit of installed capital in either sector is zero. Capital is internationally mobile, while labor can only move across sectors and not national borders. The domestic labor force grows constantly at the rate of $n$.

\[ \dot{L}(t) = n. \] (1)

The ‘hat’ notation above a variable in (1) and below denotes a rate of percentage change. The total labor force $L$ at any time is fully employed in both sectors of tradables ($L_T$) and non-tradables ($L_N$), so that

\[ L = L_T + L_N. \] (2)

The production functions for tradables ($Y_T$) and non-tradables ($Y_N$) are given in (3) and (4).

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5) In the tradition of productivity approach in determining real exchange rate are Balassa (1964), Hsieh (1982), and Yoshikawa (1990).
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\[
Y_r = \theta_r k_T^\alpha L_T^{1-\alpha} \equiv \theta_r L_T f(K_T / L_T) \equiv \theta_r L_T k_T^\alpha, \tag{3}
\]

\[
Y_N = \theta_N K_N^{\beta} L_N^{1-\beta} \equiv \theta_N L_N g(K_N / L_N) \equiv \theta_N L_N k_N^\beta. \tag{4}
\]

Here capital-labor ratios in the two sectors are denoted by \( k_T \equiv K_T / L_T \) and \( k_N \equiv K_N / L_N \). We identify the relative price of non-tradables in terms of tradables with the real exchange rate. We denote the real exchange rate as \( q \). A rise in \( q \) represents real appreciation of home currency, a fall in \( q \) real depreciation.\(^6\)

This small open economy confronts with a parametric rate of return on capital \( r \), which is represented in terms of tradable goods. Profit maximising conditions for competitive producers to employ the capital in either sector are

\[
r = \theta_r f'(k_T) = \theta_r \alpha k_T^{\alpha-1}, \tag{5}
\]

\[
r = q\theta_N g'(k_N) = q\theta_N \beta k_N^{\beta-1}. \tag{6}
\]

Equation (5) ties down \( k_T \) as a function of parameters of \( r \), \( \theta_r \) and \( \alpha \). Given \( k_T \) derived in this way, the wage \( w \) is determined by the marginal product of labor in tradable goods sector.

\[
w = \theta_r [f(k_T) - f'(k_T)k_T] = \theta_r (1-\alpha)k_T^\alpha = (1-\alpha)\theta_r \frac{1}{\alpha} (\alpha / r)^{\frac{1}{\alpha-1}}, \tag{7}
\]

\(^6\) The real exchange rate is defined as, \( q = p / e \).

Assuming that home country is a small open economy, then we can regard \( P^*_T \) as parametrically given. Then we can interpret \( q \) as the increasingly monotonic function of price of non-tradables.
Given the international price of $r$, the wage $w$ is determined wholly by factor productivity in tradables. This result rules out the possible specialization in non-tradables. In principle, the economy can specialize in the production of non-tradables, by financing its consumption of tradables out of foreign asset holdings. But we restrict our discussion to non-specialization case.

Now we are in a position to derive the long-run equilibrium real exchange rate (the relative price of non-tradables), $q$. The long-run competitive condition in the non-tradable sectors is

$$q\theta_N g(k_N) = r k_N + w.$$  \hspace{1cm} (8)

We derive $k_N$ from equation (6) as a function of $r$, $q$, and $\theta_N$.

$$k_N = (r / q \theta_N \beta)^{1/(\beta - 1)}.$$  \hspace{1cm} (6')

Substituting $k_N$ from (6') and $w$ from (7') into equation (8) yields

$$q(t) = x r^{1-\alpha} \theta_T(t)^{1-\beta} \theta_N(t)^{-1},$$  \hspace{1cm} (9)

where $x$ is a constant function of production function parameters $\alpha$ and $\beta$.

Equation (9) can be rewritten as

$$q = \frac{1 - \beta}{1 - \alpha} \theta_T - \theta_N.$$  \hspace{1cm} (9')

The stochastic version of $q(t)$ can be considered if we assume that $\theta_N$ is only random. The stochastic process $\theta_N(t)$ is governed by
where \( \kappa \) = a constant, \( z(t) \) = a random variable representing an adverse productivity shock in non-tradables, and \( \theta_N \) = the deterministic time trend in \( \theta_N \).

Then the stochastic analogue of equation (9) is written as

\[
q(t) = (x / \kappa) r^{1-\alpha} \theta_r(t) e^{1-\alpha} z(t). 
\]

Taking natural logarithms of (9’’) yields

\[
\ln q(t) = \ln(x / \kappa) + \ln r^{1-\alpha} + \frac{1-\beta}{1-\alpha} \ln \theta_r(t) - \hat{\theta}_N t + z(t). 
\]

Applying the first order Taylor expansion for the term \( \ln \theta_r(t) \) in the right hand side of equation (11), we can rewrite equation as

\[
\ln q(t) = \gamma + \mu t + z(t). 
\]

where \( \gamma = \ln(x / \kappa) + \frac{1-\beta}{1-\alpha} \ln r + \frac{1-\beta}{1-\alpha} \ln \theta_r(0) \) and \( \mu = \frac{1-\beta}{1-\alpha} \hat{\theta}_r - \hat{\theta}_N \).

Suppose discrete time version of \( z(t) \) follows first order AR (Autoregressive) process.

\[
z(t) = \phi z(t-1) + \epsilon(t). 
\]

Then (11’) and (12) imply

\[
\ln q(t) = [(1-\phi)\gamma + \mu t] + \mu(1-\phi) + \phi \ln q(t-1) + \epsilon(t). 
\]

Above process is trend stationary if \( \phi \) is less than one. On the other hand, the log exchange rate follows the random walk if \( \phi = 1 \). Then log
real exchange rate can be written as

\[ \ln q(t) = \mu + \ln q(t - 1) + \varepsilon(t). \] (14)

2.2. Empirical Implementation

Japanese real effective exchange rate (JRER) over the period January 1980–June 2013 is plotted in figure 1.\(^7\) The appreciating trend of the real yen reaches the peak at June 1995. Afterwards, the real yen exhibits a depreciating trend until financial crisis in 2007 which originated from the U.S.

To account for this pattern of trend, we specify the following univariate form for the data generating process of Japanese real effective exchange rate.

**Figure 1  Japanese Real Effective Exchange Rate  
over The Period January 1980-June 2013**

\(^7\) The data for Japanese real effective exchange rate was obtained from Bank of Japan, Monthly Economic Statistics. The indices for real EER comprise 27 countries, with data from 1964. The weighting pattern is time varying and based upon trade.
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\[
\ln q(t) = \gamma_0 + u_0 t + z(t), \quad t \leq t_0, \\
\ln q(t) = \gamma_1 + u_1 t + z(t), \quad t \geq t_0, \\
z(t) = \phi_1 L z(t) + \phi_2 L^2 z(t) + \varepsilon(t),
\]

where \( \mu_0 (\mu_1) \) is the unconditional deterministic trend in the natural logarithm of the real exchange rate, \( L \) is lag operator, and \( \varepsilon(t) \) is white noise. \( t_0 = \text{June 1995}. \)

An important issue that needs to be resolved in this data generating process is to allocate the trend in real exchange rates between stochastic and deterministic components.

Over the sample period, we examine two versions of the above data generating process, one version of which specifies the stochastic process of \( z(t) \) being stationary, and the other version of which restricts the process to have a unit root. Under the specification that \( z(t) \) process is stationary the statistical result of this over the period Jan 1980-June 1995 is summarized in table 1.

**Table 1 Real Exchange Rate under Trend Stationary Process, Sample (Adjusted): Mar 1980 to June 1995**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>( t )-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ((c))</td>
<td>4.265971</td>
<td>0.107370</td>
<td>39.73167</td>
</tr>
<tr>
<td>TREND ((\gamma))</td>
<td>0.003211</td>
<td>0.000873</td>
<td>3.677744</td>
</tr>
<tr>
<td>AR(1) ([\phi_1])</td>
<td>1.257628</td>
<td>0.070954</td>
<td>17.72460</td>
</tr>
<tr>
<td>AR(2) ([\phi_2])</td>
<td>-0.295138</td>
<td>0.071325</td>
<td>-4.137933</td>
</tr>
</tbody>
</table>

\( R \)-squared : 0.984306  
Adjusted \( R \)-squared : 0.9842044  
Inverted AR Roots : 0.95, 0.31

8) The roots of the polynomial equation \( 1 - \phi_1 L - \phi_2 L^2 = 0 \) are assumed to lie outside of the unit circle or on that.
Over the sample period Jan 1980-June 1995 the unconditionally expected trend rate of real yen appreciation is about .32% percent per month, annually about 4% and it is also statistically significant.\(^9\)

The inverted AR Roots are in the unit circle, which implies that the data fail to give strong evidence against the hypothesis that the log of the real yen rate follows a random walk process. The estimated coefficient of \( \phi_2 \) is \(-0.293633\), the \( t \)-statistic of which is \(-4.121885\), implying that \( \phi_2 \) is not zero. To treat the non-stationary case, we impose one unit root on the characteristic equation as

\[
1 - \phi_1 L - \phi_2 L^2 = 0 \Rightarrow 1 - \phi_1 - \phi_2 = 0.
\]

Under the above restriction we rewrite the data generating process as

\[
(1 - L) \ln \frac{q(t)}{q(t-1)} = \gamma (2 - \phi_1) + (\phi_1 - 1)(1 - L) \ln \frac{q(t-1)}{q(t-2)} + \varepsilon(t).
\]

Above equation is nonlinear in parameters of \( \phi_1 \) and \( \gamma \). Table 2 shows the statistical estimation result over the sample period 1980 Jan-June 1995.

**Table 2  Real Exchange Rate under Unit Root Process, Sample (Adjusted): Mar 1980 to June 1995**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>( t )-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>0.003797</td>
<td>0.002375</td>
<td>1.598890</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>1.274705</td>
<td>0.070941</td>
<td>17.96847</td>
</tr>
</tbody>
</table>

\( R \)-squared : 0.0761170  
Adjusted \( R \)-squared : 0.071041

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\(^9\) First order differencing the EQUATION in table 1 yields \( \log(q(t)) - \log(q(t-1)) = \gamma + z(t) - z(t-1) \). Ignore the process of error term, then the appreciation rate of real yen \( \frac{q(t) - q(t-1)}{q(t-1)} \) is \( \gamma \).
Table 3  Real Exchange Rate under Trend Stationary Process, Sample (Adjusted): July 1995 to June 2013

EQUATION: $\log(q(t)) = c + \gamma t + z(t)$. $z(t) = \phi_1 z(t-1) + \phi_2 z(t-2) + \epsilon(t)$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ($c$)</td>
<td>4.554645</td>
<td>0.043802</td>
<td>103.9834</td>
</tr>
<tr>
<td>TREND ($\gamma$)</td>
<td>5.77E-06</td>
<td>0.000123</td>
<td>0.04675</td>
</tr>
<tr>
<td>AR(1) [$\phi_1$]</td>
<td>1.330961</td>
<td>0.063879</td>
<td>20.83566</td>
</tr>
<tr>
<td>AR(2) [$\phi_2$]</td>
<td>–0.372642</td>
<td>0.062928</td>
<td>–5.921738</td>
</tr>
</tbody>
</table>

$R$-squared : 0.961168  
Adjusted $R$-squared : 0.960618  
Inverted AR Roots : 0.93, 0.40

In contrast to the stationary case, the statistical result of which is shown in table 1, the time trends are statistically insignificant under the specification of unit root process: The coefficient of time trend is 0.003797, the $t$-statistic of which is about 1.6, which implies that it is not significant at 5% significance level. 

The real appreciation trend of yen reaches the peak at June 1995, when its trend shifts to the direction of real depreciation until the financial crisis originated from the U.S. in 2007. Afterwards the real yen sharply appreciated. Empirical evidence of the real yen process over the sample period July 1995-June 2013 is represented in table 3 and 4. Under the specification of trend stationary real yen process the statistical result is shown in table 3. On the other hand table 4 shows the statistical result under the specification of unit root process of real yen.

Under the specification of trend stationary process of real yen the time trends are not significant. The estimated coefficient of $\phi_2$ is $–0.37262$, the $t$-statistic of which is $–5.921738$, implying that $\phi_2$ is not zero. Imposing one unit root in the data generating process as reported in table 2, we represent the statistical estimation result in table 4.
Table 4  Real Exchange Rate under Unit Root Process,  
Sample (Adjusted): July 1995 to June 2013

Non Linear Equation: \((1 - L) \ln q(t) = \gamma(2 - \varphi_1) + (\varphi_1 - 1)(1 - L) \ln q(t - 1) + \varepsilon(t)\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>-0.266675</td>
<td>0.265945</td>
<td>-1.002746</td>
</tr>
<tr>
<td>(\varphi_1)</td>
<td>1.383980</td>
<td>0.063336</td>
<td>21.85137</td>
</tr>
</tbody>
</table>

R-squared : 0.146577  
Adjusted R-squared : 0.142589

Table 5  Real Exchange Rate under Trend Stationary Process,  
Sample (Adjusted): March 1980 to June 2013

EQUATION : \(\log(q(t)) = c + \gamma t + z(t), \ z(t) = \varphi_1 z(t - 1) + \varphi_2 z(t - 2) + \varepsilon(t)\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ((c))</td>
<td>4.721360</td>
<td>0.250753</td>
<td>18.82876</td>
</tr>
<tr>
<td>TREND ((\gamma))</td>
<td>-0.000308</td>
<td>0.000558</td>
<td>-0.551119</td>
</tr>
<tr>
<td>AR(1) ([\varphi_1])</td>
<td>1.321815</td>
<td>0.047213</td>
<td>27.99664</td>
</tr>
<tr>
<td>AR(2) ([\varphi_2])</td>
<td>-0.342018</td>
<td>0.047241</td>
<td>-7.239804</td>
</tr>
</tbody>
</table>

R-squared : 0.976206  
Adjusted R-squared : 0.976026  
Inverted AR Roots : 0.97, 0.35

Similar to the stationary case reported in table 3, the coefficient of time trends are not statistically insignificant.

Finally we consider the whole sample period Jan 1980-June 2013. As in tables 1-4, we represent the statistical estimation result in table 5 under the specification of trend stationary data generating process. On the other hand, under the specification of unit root process of the data, we show the statistical estimation result in table 6.

The real yen time trend is not significant under the specification of trend stationary process.
Table 6  Real Exchange Rate under Unit Root Process, 
Sample (Adjusted): February 1980 to June 2013

Non Linear Equation: \((1-L)\ln q(t) = \gamma(2-\phi_1) + (\phi_1 - 1)(1-L)\ln q(t-1) + \varepsilon(t)\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>0.026061</td>
<td>0.181697</td>
<td>0.14343184</td>
</tr>
<tr>
<td>(\phi_1)</td>
<td>1.350085</td>
<td>0.046965</td>
<td>28.74686</td>
</tr>
</tbody>
</table>

* R-squared : 0.122239
* Adjusted R-squared : 0.120039

The estimated coefficient of \(\phi_2\) is –0.342018, the t-statistic of which is –7.229804, implying that \(\phi_2\) is not zero. Imposing one unit root in the data generating process we estimated the data generating process as in table 2 and 4, the result of which is shown in table 6.

As with the stationary case, the time trends are statistically insignificant under the specification of unit root process.

3. REAL INTEREST DIFFERENTIALS, NATURAL REAL RATES, AND THE REAL EXCHANGE RATE

3.1. Real Interest Rate, Real Exchange Rate and Relative Consumption of Tradable Goods

In section 1 we have derived the real exchange rate in terms of factor productivity parameters \(\theta_T\) and \(\theta_N\) of tradable and non-tradable goods. In deriving the real exchange rate we focus only on the production side. In this section we turn to the economy’s consumption side. This economy is a representative dynasty that grows at rate \(n\), less than \(r\). Its members have identical and homogeneous preferences. So social planner’s problem is to maximize the discounted value of current and future generations’ utility from consumption of tradables and non-tradables,
\[
\int s \left[ v \log c_T(s) + (1 - v) \log c_N(s) \right] e^{(n - \delta)x t} ds,
\]

where \( c_T \) and \( c_N \) are per capita consumption levels and the subjective discount rate \( \delta \) is assumed to exceed \( n \).

To solve the above dynamic problem, we should derive the current indirect function as

\[
\text{Max}_{c_T, c_N} \left[ v \log c_T + (1 - v) \log c_N \right], \text{ subject to } c_T + q c_N \leq c.
\]

Then we can show that indirect utility function is the function of \( c \) and \( q \) as

\[
\text{Max}_{c_T, c_N} \left[ v \log c_T + (1 - v) \log c_N \right] = v \log(vc) + (1 - v) \log[(1 - v)c / q]
\]

\[
= \log(c) + v \log v + (1 - v) \log(1 - v) + (1 - v) \log q,
\]

where \( c_T = vc, \ c_N = (1 - v)c / q \).

The dynamic per capita capital accumulating is defined as

\[
dk(t) / dt = rk(t) + w(t) - c(t). \tag{15}
\]

Given the above constraint, Hamiltonian function is defined as

\[
H = e^{(n - \delta)t} \left[ \log(c(t)) + \lambda(t) \right] [rk(t) + w(t) - c(t)]. \tag{16}
\]

The first order condition for \( c \) is

\[
1 / c(t) = \lambda(t). \tag{17}
\]

Equation of motion co-state variable is

\[\text{(10)}\] For heuristic derivation of Hamiltonian function, refer to Obstfeld’s working paper (1992) on ‘Dynamic Optimization in Continuous-Time Economic Models’.
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\[ de^{\lambda(t)} / dt = -\partial H / \partial \lambda(t), \]

\[ \Rightarrow (n - \delta)[e^{(n-\delta)t} \lambda(t)] + e^{(n-\delta)t} \cdot d \lambda(t) / dt = -e^{(n-\delta)} \lambda(t) \]

\[ \Rightarrow r \frac{d \lambda(t)}{\lambda(t)} = -r - (n - \delta). \]

Taking logarithms of the first order condition for \( c \) is

\[ -\log(c(t)) = \log(\lambda(t)). \]

Differentiating above equation with respect to time \( t \) yields as

\[ \frac{d \lambda(t)}{\lambda(t)} = -\frac{dc(t)}{c(t)}. \]

So,

\[ \dot{c} = r + n - \delta, \text{ which implies that } \dot{c}_r = r + n - \delta. \]

Using the results of demand functions for tradable and non-tradable goods in deriving the indirect utility function, we can show

\[ e_N = [(1 - v) / v]c_r / q. \]

This gives the dynamics of non-tradable good as

\[ \dot{c}_N = r + n - \delta - \hat{q}. \]

Define price index as

\[ p = v \log(1) + (1 - v) \log(q). \]
According to the price index, inflation rate is defined by

$$ \hat{p} = (1 - v) \hat{q}. \quad (23') $$

The real interest rate in this economy is just

$$ r - (1 - v) \hat{q} = r - (1 - v) \left\{ \frac{1 - \beta}{1 - \alpha} \hat{\theta}_T - \hat{\theta}_N \right\}. \quad (24) $$

Above expression says that the national real interest rates need not converge to the foreign rate. The foreign-domestic interest differentials widen as long as the factor productivity differentials between sectors persist. For example, a permanent fall in productivity growth in non-tradables entails a permanent rise in the equilibrium rate of increase in \( q \), and thus a fall in the domestic real interest rate.

The economy’s equilibrium consumption path for tradables and non-tradables is not balanced.

$$ \hat{c}_T - \hat{c}_N = \hat{q} = \frac{1 - \beta}{1 - \alpha} \hat{\theta}_T - \hat{\theta}_N. \quad (25) $$

The dynamics of both the tradable and non-tradable goods are that the relative consumption of tradable goods will rise over time if productivity growth is faster in tradables than in non-tradables and \( \beta > \alpha \), as is typical.

### 3.2. Identifying Nominal Shocks: Real Interest Parity Condition

#### 3.2.1. The model

Real exchange rate is very important in the side of consumption: Real appreciation (depreciation) of currency causes the real interest rate to decrease (increase), while it causes the relative consumption of tradables to increase (decrease). But this statement does hold when (1) home country is
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small in the sense that its interest rate is given parametrically in the world, (2) real exchange rate is determined wholly in the production side. Bearing this in mind, we are now in a position to emphasize the nominal factor in explaining the behavior of real exchange rate.\(^{11}\)

To do this, we investigate the real interest parity condition.

Uncovered nominal interest parity condition implies the uncovered real interest parity condition. Consider nominal interest parity condition as

\[
i(t) = i(t)^* + [E(e(t + 1)|I(t)) - e(t)]/e(t), \tag{26}
\]

where \(i(t)\) = Home nominal interest rate, \(i(t)^*\) = Foreign interest rate, \(e(t)\) = exchange rate, \(E\) = Expectation operator, \(I(t)\) = Information set available at time \(t\).

This equation says that any nominal interest differential between currencies must be offset by expected depreciation of the high interest currency against the low-interest currency.

Real interest parity condition is derived by subtracting expected inflation differentials between home and foreign countries \([E(\pi(t + 1)|I(t)) - E(\pi^*(t + 1)|i(t))]\) from the left and right hand side of the above parity condition yields

\[
i(t) - [E(\pi(t + 1)|I(t)) - E(\pi^*(t + 1)|I(t))] = i(t)^* - [E(\pi(t + 1)|I(t)) - E(\pi^*(t + 1)|I(t))] + [E(e(t + 1)|I(t)) - e(t)]/e(t)\]

\[
\Rightarrow [i(t) - E(\pi(t + 1)|I(t))] = [i^*(t) - E(\pi^*(t + 1)|I(t))] + [E(e(t + 1)|I(t)) - e(t)]/e(t) + E(\pi^*(t + 1)|I(t)) - E(\pi(t + 1)|I(t))\]

\[
\Rightarrow r(t) = r^*(t) + \ln E(q^{-1}(t + 1)|I(t)) - \ln q^{-1}(t), \tag{27}
\]

\[^{11}\text{Under the framework of short run sticky price in goods market, Dornbush (1976) have shown that permanent monetary expansion may cause the real exchange rates to overshoot in the short run, eventually reverting to the long run level. Short run volatility of real and nominal exchange rate may be caused by nominal socks of money supply.}\]
where, \( \ln E(q^{-1}(t+1) | I(t)) \) – \( \ln q^{-1}(t) \)
\[
=[E(e(t+1) | I(t) - e) / e] + E(\pi^*(t+1) | I(t)) - E(\pi(t+1) | I(t)),
\]
\[r(t) = \hat{i}(t) - E(\pi(t+1) | I(t)), \quad r^*(t) = \hat{i}^*(t) - E(\pi^*(t+1) | I(t)). \]

This real interest parity condition says that a rise in real interest differential \('r(t) - r^*(t)')\, holding expected log real exchange rate \( \ln E(q^{-1}(t+1) | I(t)) \), elicits a real domestic appreciation. To understand real interest parity condition more deeply, we introduce the natural level of a variable, the level consistent with market clearing in a flexible-price framework. Denote \( \hat{q}^{-1}(t) \) as flexible price counterpart of \( q^{-1}(t) \). We assume that the expected value of \( \hat{q}^{-1}(t) \) coincides with \( E(q^{-1}(t+1) | I(t)) \). The flexible price analog of above real interest parity condition (27) is

\[
\hat{r}(t) = \hat{r}^*(t) + \ln E(q^{-1}(t+1) | I(t)) - \ln \hat{q}^{-1}(t). \quad (28)
\]

By subtracting (28) from (27), we obtain the following relationship:

\[
\ln q^{-1}(t) = \ln \hat{q}^{-1}(t) + (r^*(t) - \hat{r}^*(t)) - (r(t) - \hat{r}(t)). \quad (29)
\]

Above equation says that the relative expansion of domestic money supply causes the real exchange rate to depreciate from the equilibrium real value of domestic currency, while the relative expansion of foreign money supply leads to the real appreciation from the equilibrium value of domestic currency. Here, the relative expansion of domestic money supply means that \([r(t) - \hat{r}(t)]\) decreases relative to \([r^*(t) - \hat{r}^*(t)]\), and vice versa for the relative expansion of foreign money supply.

---

12) \( q^{-1}(t) = e^{\pi^* / p} \).

\[
d \ln q^{-1}(t) / dt = \frac{dq^{-1}(t) / dt}{q^{-1}(t)} = \frac{E(q^{-1}(t + 1) | I(t)) - q^{-1}(t)}{q^{-1}(t)} = \ln E(q^{-1}(t + 1) | I(t)) - \ln q^{-1}(t).
\]

Note that
\[
d \ln q^{-1}(t) / dt = [E(e(t + 1) | I(t) - e(t)) / e(t)] + E(\pi^*(t + 1) | I(t)) - E(\pi(t + 1) | I(t)).
\]
3.2.2. Empirical implementation

Implementing equation (29) empirically is difficult because the data for flexible counterpart of real exchange rate and real interest rate are not easily constructed.

To circumvent this difficulty of empirical implementation of equation (29), we rewrite equation (27) as

\[
\ln q^{-1}(t) = \ln E(q^{-1}(t+1) \mid I(t)) - (r(t) - r^*(t)).
\]

This relationship implies that if the expected level of real exchange rate is constantly given, the currency will appreciate in real terms when the domestic-foreign real interest differential rises.

Estimating equation (27') requires us to construct the data for log expected value of the inverse of real exchange rate. To do this, we postulate that the stochastic process for \( q^{-1}(t) \) is given as

\[
q^{-1}(t) = \eta + ut + \zeta(t) \quad \text{where} \quad \zeta(t) = \phi_1 \zeta(t-1) + \phi_2 \zeta(t-2).
\]

We have already empirically shown that Japanese real exchange rate has followed unit root process since 1980. This empirical evidence allows us to impose one unit root on the stochastic process of \( \zeta(t) \). So we derive the following Autoregressive Process for \( q^{-1}(t) \).

\[
q^{-1}(t) = \mu(2 - \phi_1) + \phi_0 q^{-1}(t-1) + (1 - \phi_1) q^{-1}(t-2) + \epsilon(t).
\]

Then we can define the conditional expected value of \( q^{-1}(t+1) \) given information set available at time \( t \) as

\[
E(q^{-1}(t+1) \mid I(t)) = \mu(2 - \phi_1) + \phi_0 q^{-1}(t) + (1 - \phi_1) q^{-1}(t-1).
\]

13) Equation (28) can be written as \(-\tilde{r}(t) + \tilde{r}^*(t) + \ln E(q^{-1}(t+1) \mid I(t)) = \ln q^{-1}(t)\). Substituting this into equation (29) yields equation (27').
We are now in a position to specify econometric analogue for real interest parity condition (27').

\[
\ln q^{-1}(t) - \ln E(q^{-1}(t+1)|I(t)) = \beta(r(t) - r^*(t)). 
\]

The key testing restriction in the real interest parity condition is that the domestic currency will appreciate in real terms when the domestic-foreign real interest differential rises: In the above equation, \(d \ln q^{-1}(t)/d[r(t) - r^*(t)] = \beta = -1\).

The relative tightening of domestic money supply may cause real interest differentials between currencies to rise, which will raise the value of domestic currency in real terms while the relative expansion of domestic money supply may cause real interest differential to fall, which will depreciate the domestic currency in real terms vice versa. The mechanism hidden in this relationship is found in Dornbush (1976)'s sticky price frame work. Within this framework, monetary expansion or contraction will affect nominal and real interest rate, which will cause nominal or real exchange rate to depreciate or appreciate: Permanent monetary expansion or contraction will cause nominal or real exchange rate to overshoot or undershoot.

The data employed in estimating equation (30) are that real exchange rate is Japanese real effective exchange rate, and the foreign interest rate \(r^*(t)\) should be the (trade) weighted average foreign interest rates. Instead of using the (trade) weighted average, we used the U.S interest rate in estimating equation (33).

The real interest parity condition imposes parameter \(\beta\) being one on the above equation. The estimated empirical results of equation in (33) are presented in tables 7-9.

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14) Real interest rate \(r(t)\) and \(r^*(t)\) is calculated by: \(r(t)=\)Japan’s government bond’s yield rate with 5-year maturity-inflation rate based on consumer price index (CPI) \(r^*(t)=\)U.S. treasury bond’s yield rate with 5-year maturity- inflation rate based on consumer price index (CPI). The data source for yields and CPI is as follows. U.S. Treasury bond’s yield rate=U.S. Federal Reserve board. Japan’s government bond’s yield=Bank of Japan, U.S. CPI=U.S. Bureau of Labor statistics, Japan’s CPI= Bank of Japan.
Table 7  Real Interest Parity Condition,  
Sample (Adjusted): February 1980 to June 1995

EQUATION : \[ \ln q^{-1}(t) - \ln E(q^{-1}(t+1)| I(t)) = \beta(r(t) - r^*(t)) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>-17.41044</td>
<td>3432.443</td>
<td>-0.005072</td>
<td>0.9960</td>
</tr>
<tr>
<td>( \phi )</td>
<td>1.737547</td>
<td>52.47010</td>
<td>0.033115</td>
<td>0.9736</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-2.418909</td>
<td>2.537775</td>
<td>-0.93161</td>
<td>0.3418</td>
</tr>
</tbody>
</table>

R-squared : 0.026268  
Adjusted R-squared :0.015568

Table 8  Real Interest Parity Condition,  
Sample (Adjusted): July 1995 to June 2013

EQUATION : \[ \ln q^{-1}(t) - \ln E(q^{-1}(t+1)| I(t)) = \alpha - \beta(r(t) - r^*(t)) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>-5.871358</td>
<td>232.2188</td>
<td>-0.025284</td>
<td>0.9799</td>
</tr>
<tr>
<td>( \phi )</td>
<td>1.216686</td>
<td>31.27700</td>
<td>0.038900</td>
<td>0.9690</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.829063</td>
<td>1.859668</td>
<td>0.445812</td>
<td>0.6562</td>
</tr>
</tbody>
</table>

R-squared : 0.021750  
Adjusted R-squared :0.012565

Table 7 shows the empirical results of equation (33) over the sample period 1980-June 1995.

The coefficient of relative interest differential \((r(t) - r(t^*))\) is \(-2.418909\), the t-statistic of which is about \(-0.93161\). We can’t reject the null hypothesis of the coefficient of relative interest rate differential being 0 at 10% significance level. This implies that the relative easing or contraction of domestic money supply has no effect on the real yen behavior. There is also no strict relationship between real interest differentials and the real appreciation or depreciation of the currency of Yen.
The empirical results of equation (33) over the sample period July 1995-June 2013 is shown in table 8.

The coefficient $\beta$ of relative real interest differential $(r(t) - r(t')$) is 0.829063, the $t$-statistic of which is about 0.445812. Even though there is a positive relationship between relative real interest rate differential and real depreciation of the currency of Yen during the period July 1995-June 2013, the $t$-statistic of the coefficient $\beta$ implies that there is no significant effect of real interest differential on the real Yen. There is also no strict relationship between real interest differentials and the real depreciation or appreciation of the currency of Yen.

Over each of the sub-sample periods of 1980-1995 and 1995-2013, there is no any relationship between the relative real interest differential and the real exchange rate. This implies that Japanese relative monetary contraction or expansion had not any effect on real exchange rate in each sub sample period. In this case, there is no Japanese monetary implication on the real exchange rate.

Finally table 9 shows the empirical results of equation (24) over the sample period Jan 1980-June 2013.

The coefficient of relative interest differential $(r(t) - r(t')$) is $-1045270$, the $t$-statistic of which is about $-0.661757$. The null hypothesis of the
Japanese Yen Behavior since 1980

Coefficient of relative interest rate differential being 0 is rejected at 10% significance level. This implies that the relative easing of domestic money supply causes the real yen to depreciate. Furthermore there is a strict relationship between real interest differentials and the real appreciation or depreciation of the currency of Yen. The striking result is that contrary to the result of each of sub sample periods of 1980-1995 and 1995-2013, real interest parity condition does hold over the whole sample period Jan 1980-June 2013. As shown in Obstfeld (2011), this result is consistent with high correlation between the Japan-U.S. real interest differentials and the log real exchange rate for the period from June 1980 to July 2008: The correlation coefficient between these variables was +0.45.

Overall impression of this result is that Japanese monetary contraction or expansion relative to foreign countries, especially to the U.S. may cause Japanese Yen to appreciate or depreciate. Thus, some nominal monetary factors are influential in the behavior of real Yen besides real shocks emphasized in section 1 and section 2.1.

The policy implication of this result is that Japanese monetary policy is effective in stabilizing Japanese turbulent economy since 1980. As Obstfeld (2011) argues, real Yen depreciation trend with monetary easing of Japan after the collapse of Japanese bubble in 1995 has somewhat bolstered the shattering economy through export promotion, though renewed trade friction with the United States remains a threat. The Japanese Yen appreciated sharply late in 2008 in the midst of the global financial crisis, which once again has thrown Japan into deep recession. Facing with deep economic recession in Japan, the Bank of Japan is now pursuing monetary expansion. Given the real interest parity condition holds, the policy option taken by Central Bank when facing Economic Crisis is to increase monetary base, which might boost aggregate consumption and investment, also promoting export due to concurrent depreciation of domestic currency. The direction of current monetary policy in Japan is correct in this sense.
4. CONCLUSION

We investigated the real effective exchange rate of Japan over the sample period 1980-2013. Over the sub sample period of Jan 1980-June 1995 there is some evidence of deterministic appreciation trend of real yen about 0.33% monthly, i.e., 4% annually. But, the inverted AR roots are in the unit circle, which implies that the data fails to give strong evidence against the hypothesis that the log of the real yen rate follows a random walk process. Under the specification of unit root, the deterministic upward trend of real yen is not statistically significant. Over the sub sample period of July 1995-June 2013, the deterministic upward trend of real yen is not statistically significant under the specification of trend stationary or the unit root process of real exchange rate. The empirical result over the whole period Jan 1980-June 2013 is almost the same as the result over the sub sample period of July 1995-June 2013. Focusing on the consumption side of real economy, we have showed that small economy’s real interest rate goes down when the currency appreciates in real terms. Departing from real factors of production or consumption, we have emphasized nominal factors in influencing the real Yen behavior. To show this, we have set up the real interest parity condition and also elicited some testable restrictions. Even though we have not supported real interest parity condition over each of the sub sample periods, we have found strong evidence that real interest parity condition does hold surprisingly over the whole sample period. Strong Yen was associated with relatively higher Japanese interest rates, while weak Yen was associated relatively lower interest rate: Relative easing of Japanese domestic money supply causes the real Yen to depreciate while its relative tightened money causes the real Yen to appreciate. This implies that Japanese monetary policy is effective in stabilizing Japanese turbulent economy since 1980. Future research should be done in anatomizing real exchange rate in terms of ‘overall terms of trade’ and other real terms. Panel data analysis should be employed in econometric implementation of nominal or real interest parity condition. Of course data set should be expanded.
REFERENCES


Jin-Ock Kim · Gi-Choon Kang


